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1985 J. Phys. A: Math. Gen. 18 3071

(http://iopscience.iop.org/0305-4470/18/15/029)

J. Phys. A: Math. Gen. 18 (1985) 3071-3072. Printed in Great Britain

COMMENT

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Received 22 March 1985

Abstract. The formulae given by Brudno are generalised.

In a recent paper by Brudno (1985) some formulae for non-trivial zeros of the $3j$- and $6j$-symbols have been found. This paper ends with the statement: 'Proof that these equations constitute the total solution to the linear nontrivial problem, however, has yet to be demonstrated'. (Brudno defines the problem of finding non-trivial zeros to be linear when the sums for the calculation of the coefficients have two terms: the linear solutions are those, where both terms cancel each other.)

It may be worth noting that this problem has already been solved in my book on angular momentum (Lindner 1984)—on p 39 for the $3j$-symbol and on p 59 for the $6j$-symbol.

The zeros called linear solutions by Brudno exist for the $3j$-symbol if and only if the smallest element of the Regge symbol (Regge 1958)

$$\begin{bmatrix} -j_1+j_2+j_3 & j_1-j_2+j_3 & j_1+j_2-j_3 \\ j_1-m_1 & j_2-m_2 & j_3-m_3 \\ j_1+m_1 & j_2+m_2 & j_3+m_3 \end{bmatrix} = \begin{bmatrix} n_{11} & n_{12} & n_{13} \\ n_{21} & n_{22} & n_{23} \\ n_{31} & n_{32} & n_{33} \end{bmatrix}$$

is equal to 1 and the algebraic minor of that element is zero (e.g. if $n_{11}=1$ then $n_{22}n_{33}$ must be equal to $n_{23}n_{32}$ for a non-trivial zero).

The linear solution of the $6j$-symbol exists if and only if the smallest element of the Shelepin symbol (Shelepin 1964)

$$\begin{bmatrix} -j_1+j_2+j_3 & -j_1+j_2'+j_3' & -j_1'+j_2+j_3' & -j_1'+j_2'+j_3 \\ j_1'-j_2'+j_3 & j_1'-j_2+j_3' & j_1-j_2'+j_3' & j_1-j_2+j_3 \\ j_1'+j_2-j_3' & j_1'+j_2'-j_3 & j_1+j_2-j_3 & j_1+j_2'-j_3' \end{bmatrix} = \begin{bmatrix} n_{11} & n_{12} & n_{13} & n_{14} \\ n_{21} & n_{22} & n_{23} & n_{24} \\ n_{31} & n_{32} & n_{33} & n_{34} \end{bmatrix}$$

is equal to 1 and the product of the three other elements of that row is equal to the product of the two other elements of that column times the sum of the 'elements in the next diagonal' +2. If, e.g., $n_{11}=1$ then this sum is $n_{12}+n_{23}+n_{34}+2$.

The interested reader will find the proofs of these statements in the book mentioned (Lindner 1984). In addition if the smallest element has the value 2 instead of 1, the condition for non-trivial zeros can be found there. (In this case the sums consist of three terms.)

References

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